M12/5/FURMA/SP1/ENG/TZ0/XX

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International Baccalaureate
Baccalauréat International
Bachillerato Internacional

FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 1

Friday 4 May 2012 (afternoon)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[60 marks]*.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 11]*

(a) The set $S_1 = \{2, 4, 6, 8\}$ and $\times_{10}$ denotes multiplication modulo 10.

(i) Write down the Cayley table for $\{S_1, \times_{10}\}$.

(ii) Show that $\{S_1, \times_{10}\}$ is a group.

(iii) Show that this group is cyclic. *[8 marks]*

(b) Now consider the group $\{S_2, \times_{20}\}$ where $S_2 = \{1, 9, 11, 19\}$ and $\times_{20}$ denotes multiplication modulo 20. Giving a reason, state whether or not $\{S_1, \times_{10}\}$ and $\{S_2, \times_{20}\}$ are isomorphic. *[3 marks]*

2. *[Maximum mark: 7]*

(a) Express the number 47502 as a product of its prime factors. *[2 marks]*

(b) The positive integers M, N are such that $\gcd(M, N) = 63$ and $\operatorname{lcm}(M, N) = 47502$. Given that M is even and $M < N$, find the two possible pairs of values for M, N. *[5 marks]*

3. *[Maximum mark: 13]*

(a) By evaluating successive derivatives at $x = 0$, find the Maclaurin series for $\ln\cos x$ up to and including the term in x^4. *[8 marks]*

(b) Consider $\lim\limits_{x\to 0}\dfrac{\ln\cos x}{x^n}$, where $n \in \mathbb{R}$.

Using your result from (a), determine the set of values of n for which

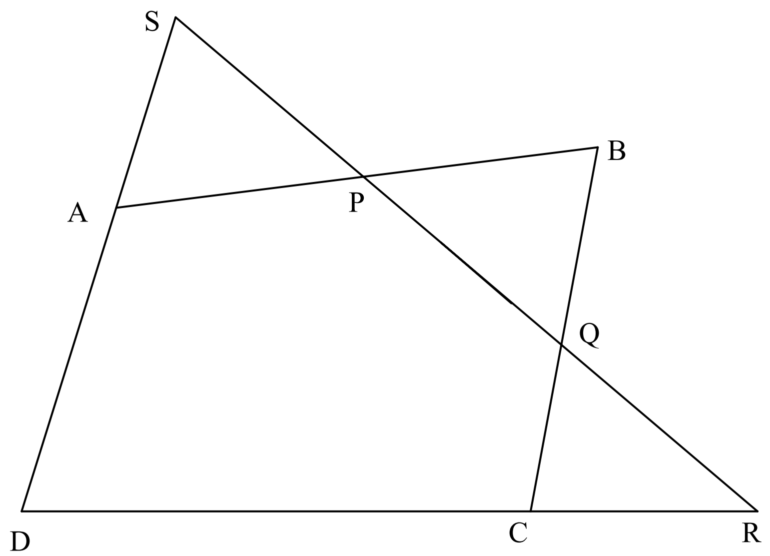
(i) the limit does not exist;

(ii) the limit is zero;

(iii) the limit is finite and non-zero, giving its value in this case. *[5 marks]*

4. *[Maximum mark: 7]*

The diagram below shows a quadrilateral ABCD and a straight line which intersects (AB), (BC), (CD), (DA) at the points P, Q, R, S respectively.



Using Menelaus' theorem, show that $\frac{\text{AP}}{\text{PB}} \frac{\text{BQ}}{\text{QC}} \frac{\text{CR}}{\text{RD}} \frac{\text{DS}}{\text{SA}} = 1$.

5. *[Maximum mark: 13]*

Bill buys two biased coins from a toy shop.

(a) The shopkeeper claims that when one of the coins is tossed, the probability of obtaining a head is 0.6. Bill wishes to test this claim by tossing the coin 250 times and counting the number of heads obtained.

(i) State suitable hypotheses for this test.

(ii) He obtains 140 heads. Find the p-value of this result and determine whether or not it supports the shopkeeper's claim at the 5 % level of significance. *[6 marks]*

(b) Bill tosses the other coin a large number of times and counts the number of heads obtained. He correctly calculates a 95 % confidence interval for the probability that when this coin is tossed, a head is obtained. This is calculated as $[0.35199, 0.44801]$ where the end-points are correct to five significant figures. Determine

(i) the number of times the coin was tossed;

(ii) the number of heads obtained. *[7 marks]*

6. *[Maximum mark: 9]*

(a) Using mathematical induction or otherwise, prove that the number $(1020)_n$, that is the number 1020 written with base n, is divisible by 3 for all values of n greater than 2. *[8 marks]*

(b) Explain briefly why the case $n = 2$ has to be excluded. *[1 mark]*